

Math 210 Portfolio

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1 Introduction

Your introduction should give the reader an overview and inform them how to read the portfolio.

2 Sums of Consecutive Integers

Theorem 1. *There is no sequence of six consecutive integers whose sum is 90.*

2.1 Current Draft

Proof. We will show that there is no sequence of six consecutive integers whose sum is 90 by showing that the sum of any six consecutive integers must be odd.

Let us start with any sequence of six consecutive integers, and let n be the first integer in the sequence. Then the sequence can be expressed as $n, n+1, n+2, n+3, n+4, n+5$. If we can show that $n + (n + 1) + (n + 2) + (n + 3) + (n + 4) + (n + 5)$ must be an odd integer, we will have proven our theorem.

Notice that, by the associative and commutative properties of the integers,

$$\begin{aligned}n + (n + 1) + (n + 2) + (n + 3) + (n + 4) + (n + 5) &= (n + n + n + n + n + n) + (1 + 2 + 3 + 4 + 5) \\ &= 6n + 15 \\ &= 6n + 14 + 1 \\ &= 2(3n + 7) + 1.\end{aligned}\tag{1}$$

Because we defined n to be an integer, 3, 7, and n are integers, and thus because the set of integers is closed under the operations of addition and multiplication, we know that $3n + 7$ is also an integer. Thus by equation (1), the sum of six consecutive integers, $n + (n + 1) + (n + 2) + (n + 3) + (n + 4) + (n + 5)$, satisfies the definition of odd integer. Since the sequence $n, n + 1, n + 2, n + 3, n + 4, n + 5$ was arbitrary, we have shown that the sum of any sequence of six consecutive integers must be odd, and in particular, since 90 is an even integer, the sum cannot be 90. \square

2.2 Previous Drafts

2.2.1 Draft 1

Proof. Let n be an integer and let $k = 3n + 7$. Then

$$\begin{aligned}n + (n + 1) + (n + 2) + (n + 3) + (n + 4) + (n + 5) &= (n + n + n + n + n + n) + (1 + 2 + 3 + 4 + 5) \\ &= 6n + 15 \\ &= 6n + 14 + 1 \\ &= 2(3n + 7) + 1 \\ &= 2k + 1.\end{aligned}\tag{2}$$

Since the sequence $n, n + 1, n + 2, n + 3, n + 4, n + 5$ is odd, it cannot be 90. \square