Chapter 1

Real number

1.1 Ordered Fields

The property of ordered fields

1.1.1 Theorem

1. A1. $\forall x, y \in \mathbb{R}$ and if $x = w$ and $y = z$, then $x + y = w + z$.
2. A2. $\forall x, y \in \mathbb{R}$, $x + y = y + x$.
3. A3. $\forall x, y, z \in \mathbb{R}$, $x + (y + z) = (x + y) + z$.
4. A4. $\exists$ unique real number $0 \ni x + 0 = x$ for all $x \in \mathbb{R}$.

1.1.2 Exercise

Q3. Let $x, y, z \in \mathbb{R}$. Prove the following.
Q3(a) $-(-x) = x$:
by M1:
Let $x = -1$, $y = -x$, then

$$\begin{aligned} x \cdot y &= (-1)(-x) \\ &= (-1 \cdot -1)x \leftarrow (from, M3) \\ &= x \end{aligned} \tag{1.1}$$

(b) $(-x) \cdot y = -(xy)$ and $(-x) \cdot (-y) = xy$:
by M3:
Let $x = -1$, $y = x$,$z = y$ then

$$\begin{aligned}(-x).y &= (-1.x).y \\ &= -(x,y) \leftarrow \text{from M3} \qquad (1.2)\end{aligned}$$

To prove second part by M3:
Let $x = -1$, $y = -x$,$z = -y$ then

$$\begin{aligned}(-x)\cdot(-y) &= (-1\cdot x)(-1\cdot y) \qquad (1.3)\\ &= (-1).x.(-1)y \\ &= xy \leftarrow \text{from M3} \qquad (1.4)\end{aligned}$$

(e) if $x \neq 0$, then $x^2 > 0$
consider $x > 0$:
Let $x = 2$:

$$\begin{aligned}x^2 &= (2)^2 \\ &= 4 \qquad (1.5)\end{aligned}$$

consider $x < 0$:
Let $x = -2$:

$$\begin{aligned}x^2 &= (-2)^2 \\ &= 4 \qquad (1.6)\end{aligned}$$