MAA

MATHEMATICAL ASSOCIATION OF AMERICA

American Mathematics Competitions

AMC 10

American Mathematics Contest 10

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 10 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded. **No copies.**
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, rulers, compass, protractors, and erasers. No calculators, smartwatches, or computing devices are allowed. No problems on the test will require the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working on the problems. You will have 75 minutes to complete the test.
9. When you finish the exam, sign your name in the space provided on the Answer Form.

The Committee on the American Mathematics Competitions (CAMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The CAMC also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

Students who score well on this AMC 10 will be invited to take the n^{th} annual American Invitational Mathematics Examination (AIME) on Thursday, March [dd], [yyyy] or Wednesday, March [dd], [yyyy]. More details about the AIME and other information are not on the back page of this test booklet.

The publication, reproduction or communication of the problems or solutions of the AMC 10 during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via copier, telephone, e-mail, World Wide Web or media of any type during this period is a violation of the competition rules.

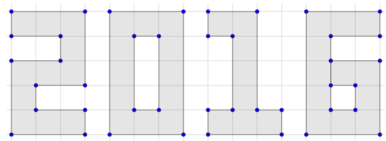
1. Find the probability you will get this question correct.

   (A) $\frac{1}{2016}$ **(B)** $\frac{1}{6}$ **(C)** $\frac{1}{5}$ **(D)** $\frac{1}{2}$ **(E)** 1

2. Find the value of $((201) \div (6^2)) + 0 - 1 \times ((62 - 0 + 1) \div 6)$.

   (A) $-\frac{59}{12}$ **(B)** $\frac{23}{6}$ **(C)** $\frac{193}{12}$ **(D)** $\frac{4447}{4}$ **(E)** $\frac{139037}{12}$

3. mathmaster2012 shaded in the number 2016 on a sheet of graph paper as shown below. Each grid square is $\frac{1}{4}$ inches wide. How much area, in square inches, did it take up?



   (A) $2\frac{11}{16}$ **(B)** $5\frac{3}{8}$ **(C)** $10\frac{3}{4}$ **(D)** $21\frac{1}{2}$ **(E)** 43

4. A picture frame in the shape of a rectangle has outer dimensions 20 by 16. The border between the picture, which also in the shape of a rectangle, and the edges of the frame is always 1 unit wide. What is the area of the border?

   (A) 68 **(B)** 72 **(C)** 74 **(D)** 76 **(E)** 78

5. There are 60 seconds in a minute, 60 minutes in an hour, 24 hours in a day, and 7 days in a week. How many seconds are in $\frac{1}{2016}$ of a week?

   (A) 150 **(B)** 200 **(C)** 240 **(D)** 300 **(E)** 360

6. A fat snorlax ate 21 hamburgers and 20 hotdogs. Another fat snorlax ate 16 hamburgers and 42 hotdogs. All the hamburgers had the same amount of calories as each other and all the hotdogs had the same amount of calories as each other. Given that both snorlaxes had the same amount of calories, how many times the amount of calories of a hotdog does a hamburger have?

   (A) $4\frac{2}{5}$ **(B)** $4\frac{1}{2}$ **(C)** $4\frac{3}{5}$ **(D)** 5 **(E)** $5\frac{1}{4}$

7. mathmaster2012 has two coupons. Coupon A gives \$20 off a purchase, while coupon B gives 16% off a purchase. When coupons are combined, discounts are taken in succession. (For example, if coupon B is used after coupon A, then 16% is taken off the remaining price after coupon A is applied.) mathmaster2012 wants to buy an item that costs \$100. Which coupon should he apply first, and how much less money would he pay than if he applied the other coupon first?

   (A) A, \$5.20 **(B)** A, \$3.20 **(C)** B, \$3.20 **(D)** B, \$5.20

   (E) He will pay the same amount no matter what.

8. The area of one square is 2016% more than the area of another. Its side length is p% more than a side length of the other square. Find p.

   (A) 340 **(B)** 360 **(C)** 440 **(D)** 449 **(E)** 460

9. mathmaster2012 is playing a video game! He currently has 2016 health. At each turn, he can either gain 20 health or lose 16 health. What is the minimum number of turns he must take to reach exactly 9001 health?

   (A) 349 **(B)** 350 **(C)** 351 **(D)** 352 **(E)** It is impossible

10. 2016 has 36 factors, and 36 is a perfect square. What is the largest number less than 2016 whose number of factors is also a perfect square?

(A) 2010 **(B)** 2011 **(C)** 2012 **(D)** 2013 **(E)** 2015

11. For nonzero real numbers a and b, $\frac{a^2+b^2}{ab} = 2016$. Evaluate $\frac{(a+b)^2}{a^2+b^2}$.

(A) 1 **(B)** $\frac{2017}{2016}$ **(C)** $\frac{1009}{1008}$ **(D)** 2017 **(E)** 2018

12. Find the remainder when the number of positive 4-digit integers $n \neq 2016$ containing each the digits 2, 0, 1, and 6 exactly once such that $gcd(2016, n) \neq 1$ is divided by 5.

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

13. A rectangle is inscribed in a circle that has a radius of 1 and an area that is 2016 times the area of the rectangle. Find the perimeter of the rectangle.

(A) $\sqrt{4+\frac{\pi}{1008}}$ **(B)** $\sqrt{4+\frac{\pi}{504}}$ **(C)** $\sqrt{4+\frac{\pi}{252}}$ **(D)** $\sqrt{16+\frac{\pi}{504}}$ **(E)** $\sqrt{16+\frac{\pi}{252}}$

14. mathmaster2012 writes the positive integers from 1 through 24 on a whiteboard. A *move* consists of erasing two numbers on the whiteboard, a_1 and a_2, and writing $\sqrt{a_1^2+a_2^2}$. qkxwsm makes moves until the whiteboard only has one number remaining. Find its maximum possible value.

(A) 48 **(B)** 69 **(C)** 70 **(D)** 71 **(E)** 72

15. Define a positive integer to be *geometic* if and only if all its digits are distinct and nonzero and they form a geometric sequence in order. For example, the numbers 1, 16, and 421 are geometric, while the numbers 11 and 400 are not. Find the remainder when the number of geometric positive integers is divided by 5.

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

16. Find the number of ordered pairs of integers (x, y) that satisfy

$$x^2 + 20x = 16y^2$$

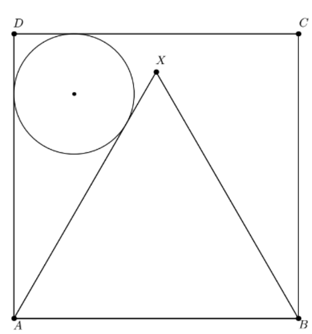
(A) 2 **(B)** 3 **(C)** 4 **(D)** 6 **(E)** Infinitely many

17. Convex non-self-intersecting quadrilateral $ABCD$ has integer side lengths and $\angle ABC = \angle ACD = 90°$. Given that $AB = 20$ and $BC = 16$, find the sum of all possible perimeters of $ABCD$.

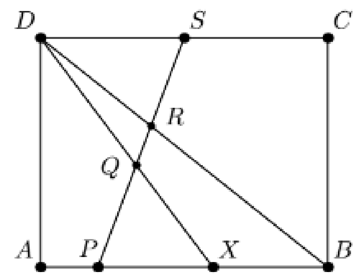
(A) 574 **(B)** 682 **(C)** 1271 **(D)** 1374 **(E)** 1451

18. Square $ABCD$ has side length 1. An equilateral triangle ABX has X inside $ABCD$. A circle fully contained inside square $ABCD$ is tangent to AX, AD, and DC. Find its radius.

(A) $3\sqrt{6}+4\sqrt{3}-5\sqrt{2}-7$ **(B)** $\frac{3-\sqrt{3}}{6}$ **(C)** $2-\sqrt{3}$ **(D)** $\frac{\sqrt{3}-1}{2}$ **(E)** $3\sqrt{6}-4\sqrt{3}+5\sqrt{2}-7$



19. Rectangle $ABCD$ has $AB = 20$ and $AD = 16$. Point X on side AB has $XD = 20$. Point P on side AB has $BP = 16$. Point S is the midpoint of side CD. Segment PS intersects DX at Q and intersects DB at R. Find $\frac{RS}{PQ}$.



(A) $\frac{25}{32}$ **(B)** $\frac{27}{32}$ **(C)** $\frac{100}{117}$ **(D)** $\frac{56}{65}$ **(E)** $\frac{45}{52}$

20. mathmaster2012 noticed that 2015 has only one 0 digit when expressed in binary. How many positive integers less than 2016 (including 2015) have only one 0 digit when expressed in binary?

(A) 39 **(B)** 40 **(C)** 49 **(D)** 50 **(E)** 51

21. The ratio between the areas of the largest semicircle and the largest circle that can be inscribed in a square is equal to

(A) $\frac{1}{2}$ **(B)** $2-\sqrt{2}$ **(C)** $\frac{\sqrt{2}+1}{4}$ **(D)** $12-8\sqrt{2}$ **(E)** $\frac{3+2\sqrt{2}}{8}$

22. Find the number of sets of distinct positive integers $\{a, b, c\}$ such that $lcm(a, b, c) = 2016$.

(A) 1852 **(B)** 1853 **(C)** 1935 **(D)** 1996 **(E)** 12103

23. Points A and B lie outside circle ω such that segment AB intersects ω at P and Q, where P is between A and Q. The tangent to ω from B intersects ω at C. Segment CA intersects ω at point M and C. Given that $BC = 20$, $AM = 16$, and $AP = BQ = x$, then the possible values of x are in the range (a, b). Find a.

(A) 9 **(B)** $\frac{61-\sqrt{521}}{4}$ **(C)** $\frac{45-15\sqrt{2}}{2}$ **(D)** 12 **(E)** $\frac{45-5\sqrt{17}}{2}$

24. Every vertex of a regular octahedron is colored red, blue, or green. Find the probability that none of the faces of the octahedron have all of its vertices the same color.

(A) $\frac{13}{27}$ **(B)** $\frac{122}{243}$ **(C)** $\frac{124}{243}$ **(D)** $\frac{14}{27}$ **(E)** $\frac{16}{27}$

25. A circle is said to *minimize* a set of points if it is a circle with minimal radius such that all the points in the set are inside or on it. 12 points are equally spaced on circle ω. A set of 4 out of these 12 points is chosen at random. Find the probability that ω minimizes this set.

(A) $\frac{17}{33}$ **(B)** $\frac{25}{33}$ **(C)** $\frac{149}{165}$ **(D)** $\frac{97}{99}$ **(E)** 1