1.1.6. Let $a, b, c \in \mathbb{Z}$ with $b, c > 0$. Suppose that when a is divided by b, the quotient is q and the remainder is r; when q is divided by c, the quotient is k and the remainder is t. Prove that when a is divided by bc, the quotient is also k.

Proof. Since a is divided by b,he quotient is q and the remainder is r. So a=bq+r, $0 < r < b$. Since q is divided by c, the quotient is k and the remainder is t. Thus, q=ck+t, $0 < t < c$. Put q=ck+t into a=bq+r, then we get a=bck+bt+r. So when a is divided by bc, the quotient is also k. □

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1.1.10. Let n be a positive integer and $a, c \in \mathbb{Z}$. Prove that: a and c leave the same remainder when divided by $n \iff \exists k \in \mathbb{Z}$ such that $a - c = nk$.

Proof. "sufficiency" Since a and c leave the same remainder when divided by n. Suppose a=nx+r, $0 < r < n$, $x \in \mathbb{Z}$. c=ny+r, $0 < r < n$, $y \in \mathbb{Z}$. So a-c=n(x-y). Since $x, y \in \mathbb{Z}$. Therefore, $x - y \in \mathbb{Z}$. Assume x-y=k, $k \in \mathbb{Z}$. So a-c=nk, and $\exists k \in \mathbb{Z}$ such that ac=nk. "necessity" Suppose a=nx+p, $0 < p < n$, $x \in \mathbb{Z}$. c=ny+q, $0 < q < n$, $y \in \mathbb{Z}$. So a-c=n(x-y)+(p-q), $x - y \in \mathbb{Z}$. Since $\exists k \in \mathbb{Z}$ such that ac=nk. Therefore p-q=0, So p=q, a and c leave the same remainder when divided by n. □

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