Hooke's Law
PHYS& 221

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Contents

1 Objective

The purpose of this lab is to determine the spring constant of a given spring. This spring constant is given by the relation between the force exerted on the spring and the distance the spring is either stretched or compressed. This relationship is given through Hooke's law which we are going to get a better understanding of throughout this lab.

1.1 Definitions

Definition 1.1. The Spring Constant: The spring constant is given by

$k = \frac{m\vec{\mathbf{g}}}{\vec{\mathbf{r}}}$,

where k is the spring constant, m is the mass, and $\vec{\mathbf{g}}$ is the acceleration due to gravity, and $\vec{\mathbf{r}}$ is the displacement.

Definition 1.2. Displacement is a vector quantity measuring the shortest path connecting two points. $\vec{\mathbf{r}}$ is the symbol for displacement. The SI unit for displacement is the meter [m].

Definition 1.3. The standard acceleration due to gravity is the nominal gravitational acceleration of an object in a vacuum near the surface of the Earth. It is defined by standard as $9.806\,65\ \mathrm{m/s^2}$. In this report, it is approximated to $9.81 \pm 0.01\ \mathrm{m/s^2}$ and its symbol is $\vec{\mathbf{g}}$.

Definition 1.4. Corrected Sample Standard Deviation: The corrected sample standard deviation, σ, is used to quantify the amount of dispersion in a set of data values, and is written:

$$\sigma = \sqrt{\tfrac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

2 Description and Theory

2.1 Principles and Theories Used to Get the Result

The principle we use throughout this lab is Hooke's law. Hooke's law is stated by F= -kx. This means that the force exerted on a spring is inversely related to the product of the spring constant and the displacement in meters. This occurs because a spring is a conservative force, a spring when stretched or compressed stores energy so that it may return to its state of equilibrium once it is released from the position it is being held at.

2.2 Derivation of Equations Used in Report

Force Applied to Mass:

$$F = ma \tag{1}$$

We know that the force being applied to the mass is equal to the acceleration being applied to the mass.

$$F = mg \tag{2}$$

In our case the acceleration is due to gravity so we use gravity in our equation.
Hooke's Law (see Force derivation):

$$F = -kr \tag{3}$$

Hooke's law was given to us throughout our studies but we are going to derive how the spring constant relates to force and displacement.

$$F_{net} = mg - kr \tag{4}$$

The only forces acting on the weights are the force of the mass multiplied by gravity and the spring constant multiplied by the displacement (which is opposing the direction of motion).

$$0 = mg - kr \tag{5}$$

At equilibrium, our weights are not in motion, so we know that the difference between mass multiplied by gravity and the spring force multiplied by displacement is zero.

$$mg = kr \tag{6}$$

From there, we can set the mass multiplied by gravity and the spring constant multiplied by displacement equal to each other. F=kr From our derivation of the force applied to the mass, we know that F=mg, so we can simply plug that into the equation.

$$\frac{F}{r} = k \tag{7}$$

We divided both sides by x which gives us the spring constant as the ratio between force divided by displacement.

Master Spring Constant Equation: (This will be used to calculate percent impacts later)

$$mg = kr \tag{8}$$

Remark. Our fully written master spring constant equation is our expanded force which is set equal to Hooke's Law.

$$\frac{mg}{r} = k$$

We then divide both sides by the displacement which gives us final equation for our spring constant.

2.3 Procedure

a. Determine the masses by weighing the mass on the scale.

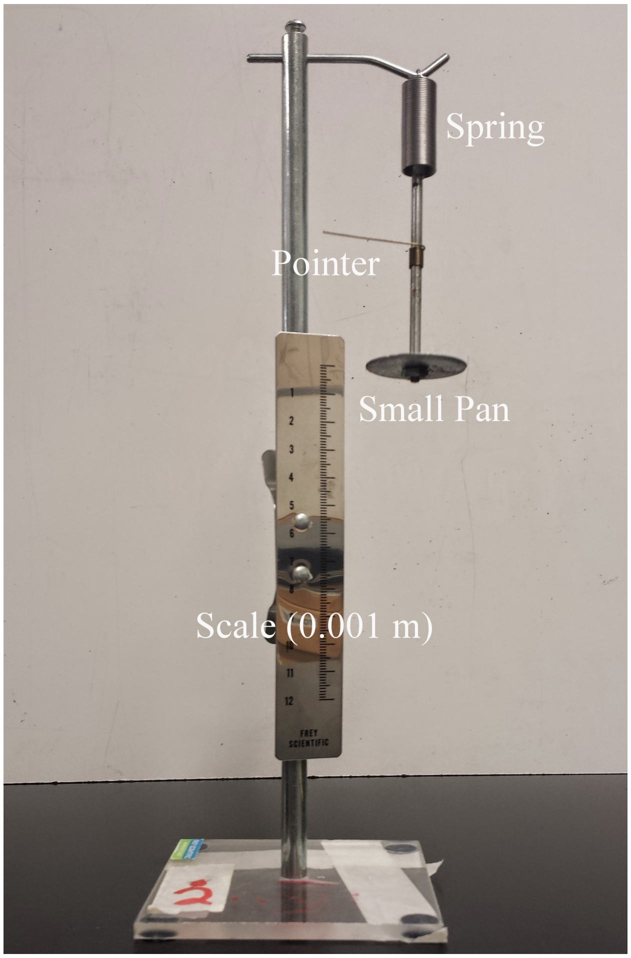
b. Next attach the masses to the spring on the small pan.

c. The pointer of the spring will be scaled at zero, then measure the position of the end of the spring after the mass has been attached by looking at the pointer. (scale unit: 0.001 m)

d. Continue weighing and measuring by increasing the mass attached to the spring to 50g, 70g, 100g, 120g, 10g, 200g, and 220g. Then measure the corresponding position of the spring for each mass.

e. All of member in the group will experiment the lab individually. Observe the change in position of the spring and record the experiment.

2.4 Apparatus



Apparatus 1: The scale was positioned so as to have the pointer, which is located between the small pan and the spring, pointing at 0.00 m. Weights of differing masses were then loaded one by one onto the small pan hanging from the spring. The scale was then used to measure the distance the pointer moved due to the weight in the pan. This distance was then recorded in the ”Displacement” column of Data Table 2 (see Section 3).

3 Experimental Data

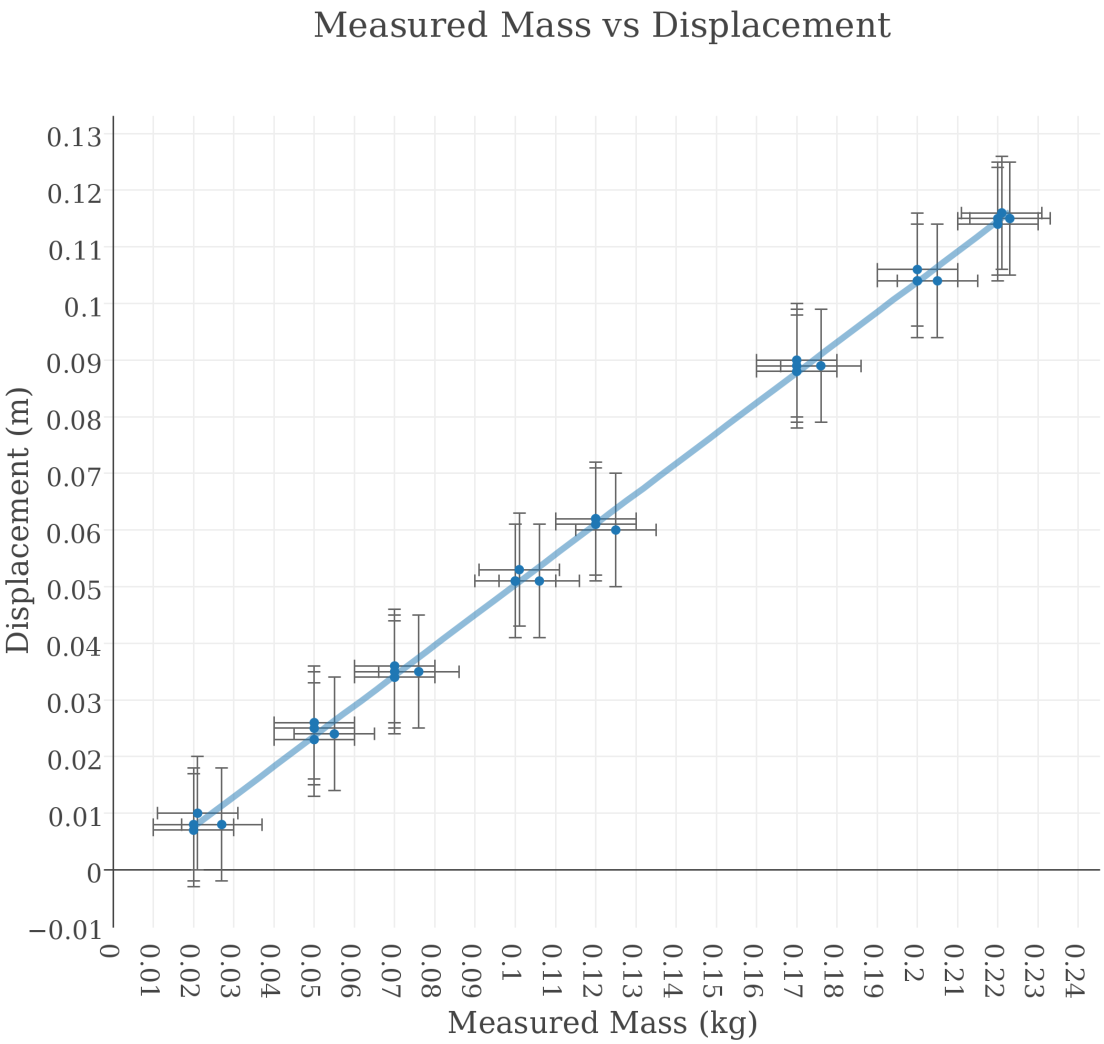
Data Table 1:
"Measured Mass" is the only measurement recorded in this table.
The spring force is derived using $F_s = -kx$ (see Table 2)

Nominal Mass (kg)	**Name**	**Measured Mass (kg)**	**Spring Force (N/m)**	σ **(kg)**
0.020	Amezola	0.020	-0.020	0.003
	Lai	0.021	-0.021	
	Mumphrey	0.027	-0.027	
	Tran	0.020	-0.020	
0.050	Amezola	0.050	-0.050	0.003
	Lai	0.050	-0.050	
	Mumphrey	0.055	-0.055	
	Tran	0.050	-0.050	
0.070	Amezola	0.070	-0.070	0.003
	Lai	0.070	-0.070	
	Mumphrey	0.076	-0.076	
	Tran	0.070	-0.070	
0.100	Amezola	0.100	-0.100	0.003
	Lai	0.101	-0.101	
	Mumphrey	0.106	-0.106	
	Tran	0.100	-0.100	
0.120	Amezola	0.120	-0.120	0.003
	Lai	0.120	-0.120	
	Mumphrey	0.125	-0.125	
	Tran	0.120	-0.120	
0.170	Amezola	0.170	-0.170	0.003
	Lai	0.170	-0.170	
	Mumphrey	0.176	-0.176	
	Tran	0.170	-0.170	
0.200	Amezola	0.200	-0.200	0.002
	Lai	0.200	-0.200	
	Mumphrey	0.205	-0.205	
	Tran	0.200	-0.200	
0.220	Amezola	0.220	-0.220	0.001
	Lai	0.221	-0.221	
	Mumphrey	0.223	-0.223	
	Tran	0.220	-0.220	

Data Table 2:
The measurement of spring displacement due the weight of a mass.

Nominal Mass (kg)	**Name**	**Displacement (m)**	**Average Displacement (m)**	σ **(m)**
0.020	Amezola	0.007	0.008	0.001
	Lai	0.010		
	Mumphrey	0.008		
	Tran	0.008		
0.050	Amezola	0.023	0.025	0.001
	Lai	0.026		
	Mumphrey	0.024		
	Tran	0.025		
0.070	Amezola	0.035	0.035	0.001
	Lai	0.036		
	Mumphrey	0.035		
	Tran	0.034		
0.100	Amezola	0.051	0.052	0.001
	Lai	0.053		
	Mumphrey	0.051		
	Tran	0.051		
0.120	Amezola	0.061	0.061	0.001
	Lai	0.062		
	Mumphrey	0.060		
	Tran	0.062		
0.170	Amezola	0.089	0.089	0.001
	Lai	0.090		
	Mumphrey	0.089		
	Tran	0.088		
0.200	Amezola	0.104	0.105	0.001
	Lai	0.106		
	Mumphrey	0.104		
	Tran	0.104		
0.220	Amezola	0.115	0.115	0.001
	Lai	0.116		
	Mumphrey	0.115		
	Tran	0.114		

4 Graphs



Graph 1: The slope of this regression line is proportional to the average spring constant k.

5 Calculations

To calculate the spring, k, we use the given equation $k = \frac{mg}{r}$. Let x equal the sample mean vector for the measured quantities of x as recorded in Table 2 (Section 3), m equal the sample mean vector for the measured quantities of m as recorded in Table 1 (Section 3), and g equal the standard acceleration of gravity, g, 9.81 m/s^2.

$$k = \frac{mg}{r}$$

$$k = \frac{0.120\,187\,5 \ \text{kg} * 9.81 \ \text{m/s}^2}{0.059\,419\,354 \ \text{m}}$$

$$k = 19.8\,\text{N/m}$$

The following formula will be used to calculate the degree of error associated with each quantity in the Master Equation above.

m: The error in m is taken to be $\delta m = \pm 0.002\,828\,427\,125\,\text{kg}$

$$k = \frac{(0.120\,187\,5 \ \text{kg} \pm 0.002\,828\,427\,125 \ \text{kg}) * 9.81 \ \text{m/s}^2}{0.059\,419\,354 \ \text{m}} = 21.4\,\text{N/m}$$

$$\% \ impact \ = \ \frac{19.84268221 - 21.38957928}{19.84268221} \times 100\% = 7.80\%$$

r: The error in r is taken to be $\delta r = \pm 0.007\,681\,145\,748 \ \text{m}$

$$k = \frac{0.120\,187\,5 \ \text{kg} * 9.81 \ \text{m/s}^2}{(0.059\,419\,354 \ \text{m} \pm 0.007\,681\,145\,748 \ \text{m})} = 17.6\,\text{N/m}$$

$$\% \ impact \ = \ \frac{19.84268221 - 17.57124581}{19.84268221} \times 100\% = 11.4\%$$

g: The error in m is taken to be $\delta g = \pm 0.01\,\text{m/s}^2$

$$k = \frac{0.120\,187\,5 \ \text{kg} * (9.81 \ \text{m/s}^2 \pm 0.01 \ \text{m/s}^2}{0.059\,419\,354 \ \text{m}} = 19.9\,\text{N/m}$$

$$\% \ impact \ = \ \frac{19.84268221 - 19.8629092}{19.84268221} \times 100\% = 0.104\%$$

6 Results and Conclusions

6.1 Results

Our spring constant which we got from averaging the slopes of our Force vs Displacement graphs (see graphs section) was calculated to be:

$$19.8 \pm 2.2 \text{ N/m}$$

6.2 Discussion of Experimental Uncertainty

6.2.1 Systematic Errors

The spring may not be in good condition; therefore it will also make the spring constant is different when we scale different masses. The systematic error could be happen when we use a centimeter stick with zero errors. This error could make the reading of the result become higher or lower than the real result. When we observe and read the result, our eyes may not perpendicular to the pointer of the spring. This error could make the result become higher or lower than the real result.

6.2.2 Random Errors

The uncertainty of m was estimated to be $\pm 0.0686\,\text{kg}$. This error could cause an uncertainty of about 7.80%. The uncertainty of x was estimated to be $\pm 0.0361\,\text{m}$. This error could cause an uncertainty of about 11.4% The uncertainty of g was estimated to be $\pm 0.01\,\text{m/s}^2$. This error could cause an uncertainty of about 0.102%

6.2.3 General Error Discussion

Throughout the whole experiment the biggest problem was the oscillation of the spring when we placed the mass on the pan. This made trying to read the scale really hard since it would oscillate between a wide range of values. We used a pen to reduce the oscillation which in turn would reduce our errors. It wasn't until the last 5 conditions where we found that we could slowly lower the masses until it reached the max stretching of the spring and that we could read the values with ALMOST NO oscillation at all, which reduced our errors almost completely. The fact that we cannot be 100% sure that the zero we had chosen was precisely zero is another cause for error, but that error would be constant throughout the whole experiment and would then be considered an systematic error.