Proposition: A number is divisible by 4 if and only if its last 2 digits are divisible by 4.

Proof. Let $n \in \mathbb{Z}$.
Assume n is divisible by 4, we want to show that the last two digits of n are divisible by 4. By the definition of divides, $n = 4b$ for some $b \in \mathbb{Z}$. n can also be expressed in the form $n = a_k * (100)^k + a_{k-1} * (100)^{k-1} + ... + a_0 * (100)^0$, where a_k is a two digit integer whose digits correspond to the digits of n. For example, 123456 can be expressed as $12 * 100^2 + 34 * 100^1 + 56 * 100^0$. By substitution, $a_k * (100)^k + a_{k-1} * (100)^{k-1} + ... + a_0 = 4b$. Rearranging the left side gives us $n = a_k * 100 * (100)^{k-1} + a_{k-1} * 100 * (100)^{k-2} + ... + a_1 * 100 + a_0 = 4(a_k * 25 * (100)^{k-1} + a_k * 25 * (100)^{k-2} + ... + a_1 * 25) + a_0 = 4b$. Consequently $4c + a_0 = 4b$ for the number $c = a_k * 25 * (100)^{k-1} + a_{k-1} * 25 * (100)^{k-2} + ... + a_1 * 25$, which is an integer under the closure properties of the integers. Rearranging once more gives $a_0 = 4(b - c) = 4d$ for the number $d = b - c$ which is an integer under the closure properties of the integers. Thus $4|a_0$ by the definition of divides. Recall that a_0 was a 2 digit integer whose digits were the last two digits of n. Therefore, the last 2 digits of n are divisible by 4 as desired.

Conversely, assume the last 2 digits of n are divisible by 4, we want to show that n is divisible by 4. Just as before, n can be represented as $n = a_k * (100)^k + a_{k-1} * (100)^{k-1} + ... + a_0 * (100)^0 = 4(a_k * 25 * (100)^{k-1} + a_{k-1} * 25 * (100)^{k-2} + ... + a_1 * 25) + a_0$. a_0 still represents the last 2 digits of n, and is therefore divisible by 4 based on the hypothesis, allowing us to express it as $a_0 = 4e$ for some integer e by the definition of divides. Using substitution, $n = 4(a_k * 25 * (100)^{k-1} + a_{k-1} * 25 * (100)^{k-2} + ... + a_1 * 25) + 4e$. Consequently, $n = 4f$ for the number $f = a_k * 25 * (100)^{k-1} + a_{k-1} * 25 * (100)^{k-2} + ... + a_1 * 25 + e$, which is an integer under the closure properties of the integers. Therefore, n is divisible by 4 as desired. ■